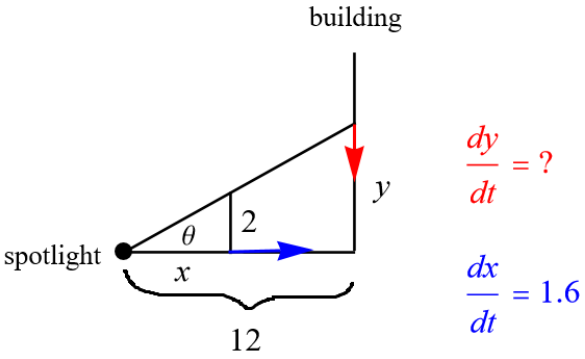
Exercise 18

A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

Solution

Draw a schematic of the spotlight, the man, and the building. Let x be his distance from the spotlight, and let y be the height of the shadow on the building wall. The derivative of a distance with respect to time is a speed.



Use trigonometry to relate x and y.

$$\tan\theta = \frac{2}{x} = \frac{y}{12}$$

Solve for y.

$$y = \frac{24}{x}$$

Differentiate both sides with respect to time by using the chain rule.

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{24}{x}\right)$$

$$= \left(-\frac{24}{x^2}\right) \cdot \frac{dx}{dt}$$

When the man is 4 m from the building, he is 8 m from the spotlight.

$$\left.\frac{dy}{dt}\right|_{x=8} = \left(-\frac{24}{8^2}\right) \cdot (1.6) = -0.6.$$

Therefore, when the man is 4 m from the building, the shadow is decreasing by 0.6 meters per second.